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therefore

$$[(r^2 - s^2)^2 - (2rs)^2]^2 + [4rs(r^2 - s^2)]^2 = (r^2 + s^2)^4,$$

where r and s may be any numbers prime to each other, one odd and the other even.

Let $r = 2$, $s = 1$, and we have

$$7^2 + 24^2 = 625 = 25^2 = 5^4.$$

Let $r = 3$, $s = 2$; then we get

$$119^2 + 120^2 = 169^2 = 13^4.$$

Let $r = 4$, $s = 1$, and we find

$$161^2 + 240^2 = 289^2 = 17^4.$$

203. Proposed by R. D. CARMICHAEL, Indiana University.

Find solutions in integers of the equation

$$2x^2 + 1 = 3y^2. \quad (1)$$

I. SOLUTION BY E. E. WHITFORD, New York City.

Let $2x = z$. Then

$$z^2 - 6y^2 = -2. \quad (2)$$

By inspection the solution in smallest positive integers is $z_1 = 2$, $y_1 = 1$. The problem will not lose in generality if the solution be limited to positive integers. To represent concisely all the positive solutions without exception and without repetition I have derived the following formula:

$$z + y\sqrt{6} = \frac{(z_1 + y_1\sqrt{6})^{2k+1}}{2^k},$$

where

$$z_1 = 2, \quad y_1 = 1, \quad k = 0, 1, 2, 3, \dots$$

Therefore

$$z = 2, \quad 22, \quad 218, \quad 2158, \quad \dots,$$

$$y = 1, \quad 9, \quad 89, \quad 881, \quad \dots$$

Since the values of z are even each set gives a solution of equation (1).

$$x = 1, \quad 11, \quad 109, \quad 1079, \quad 10681, \quad \dots,$$

$$y = 1, \quad 9, \quad 89, \quad 881, \quad 8721, \quad \dots$$

These results were obtained by Euler in his "Algebra," by de la Roche in his "Larismetique" (1520), who copied from the "Triparty" of Chuquet (1484); and probably by nearly everyone who tried, before decimal fractions came into common use, to find the approximate value of the square root of 6.

Equation (2) is a slightly generalized form of the Pell equation $x^2 - Ay^2 = 1$.

The Pell equation can be made an aid to the solution of all indeterminate equations of the second degree in two unknowns.*

II. SOLUTION BY RICHARD MORRIS, Rutgers College.

This equation is of the form $Dx^2 - Cy^2 = -H$ (see Chrystal's *Algebra*, Part II, page 450) and the integral values of x and y are dependent upon the expression of \sqrt{CD}/D as a continued fraction, the value of x being equal to the numerator and that of y to the denominator of those convergents, where

$$(-1)^n M_n = -H.$$

Let n denote the number of the convergent, A_n the quotients, p_n/q_n the terms of the convergent, and M_n the $(n+1)$ th rational divisor, and arrange these quantities in column form to exhibit the solutions. There will be an infinite number of solutions since $(-1)^n M_n = -1$ for all the odd convergents.

N	A_n	p_n	q_n	M_n
1	1	1	1	1
2	4	5	4	2
3	2	11	9	1
4	4	49	40	2
5	2	109	89	1
6	4	485	396	2
7	2	1,079	881	1
8	4	4,801	3,920	2
9	2	10,681	8,721	1

Thus $x = 1, y = 1; x = 11, y = 9; x = 109, y = 89$, etc., are integral solutions.

Also solved by C. E. GITHENS.

MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

Note.—This department is designed to furnish a forum for the discussion of live questions in the teaching of collegiate mathematics and of difficulties actually encountered by any one who may desire the reaction of other workers in the same field. It is believed that much benefit may accrue from the interchange of ideas along these lines, and it is hoped that a widespread interest will develop in maintaining the department.

EDITORS.

QUESTION.

12. In view of the notation used by Professor Slobin in his "Note on Certain Algebraic Equations" in the April issue, pages 113–115, a discussion would be desirable as to the best notation for complex roots in general, and in particular for eliminating the conspicuous ambiguities introduced by the notation above cited.

REPLIES.

7. What place should be given to the history of mathematics in courses for prospective high school teachers, and why?

* For an account of such solutions see "The Pell Equation," E. E. Whitford, *Publications of the College of the City of New York*, 1912, pages 60, 64, ff.